Rapid Calculation of Nonlinear Wave-Wave Interaction in Wave-Action Balance Equation

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1. INTRODUCTION

Calculation of the nonlinear wave-wave interaction in ocean waves has been studied since the early 1960s (Hasselmann, 1962). Because the exact solution of the wave-wave interaction requires solving a 6-dimensional integral (Resio and Tracy, 1982), many approximation methods have been developed to expedite the computation. However, none of the approximation methods appears to be practical for coastal and oceanic applications because they lack either computational efficiency or accuracy. The present study extends the theory to improve the formula derived by Jenkins and Phillips (2001) to explicitly express the nonlinear transfer of wave energy density and calculate directly in the wave-action balance equation to reduce computational time. The formula is a second-order diffusion operator of general isotropic form that conserves wave energy, momentum, and wave action. It captures the essential nonlinear wave-wave interaction feature that the nonlinear energy exchange is directed mainly from high to low frequencies while total wave energy is conserved. By implementing the new nonlinear energy transfer formula in the wave-action balance equation, the nonlinear wave-wave interaction can be rapidly and accurately calculated in the same routine solving the wave-action equation, no additional integration required, for ocean wave generation, dissipation, and propagation, and their transformation through coastal regions.

2. THEORY

Based on the method of Jenkins and Phillips (2001), retaining only the primary terms in the frequency, σ, and direction, θ, domains, the nonlinear wave energy transfer, S_{nl}, can be expressed as

$$S_{nl} = a\frac{\partial F}{\partial \sigma} + b\frac{\partial^2 F}{\partial \theta^2}$$

where $a = \frac{1}{2n^2}[1+(2n-1)^2 \cosh 2kh]-1$ is a function of kh (wave number, k, times depth, h), $b = \frac{a}{n\sigma}$,

$n = \frac{1}{2} + \frac{kh}{\sinh kh}$, $F = k^3\sigma^5 \frac{n^4}{(2\pi)^2 g}\left[(\frac{\sigma_m}{\sigma})^4 E\right]^3$, and $E = E(\sigma,\theta)$ is the directional spectrum.

Figure 1 shows the significance of the proportionality values a and $b\sigma$ as functions of kh. Non-linear wave-wave interaction initially occurs in deep water, increases in significance in intermediate depth, and gradually diminishes as waves are transformed into shallow water. The effect of the interaction on wave evolution is greater over long fetches in a large ocean domain and less in a local coastal region.

Figure 2 shows the comparison of directionally integrated S_{nl} and exact computations in the example of Hasselmann et al. (1985) for a JONSWAP spectrum with peakedness factor γ=5. The rapid calculation of S_{nl} is carried out by a two-dimensional spectral wave model in the Coastal Modeling System (http://cirp.usace.army.mil/products) at the U.S. Army Engineer Research and Development Center.

3. EXAMPLE OBSERVATIONS

In applications, nonlinear wave energy transfer is more significant for locations with wave generation (Resio and Tracy, 1982). Numerical simulations of deep-water wave generation, propagation, and transformations to the coast at bay and near inlet jetties were conducted for the north-central Gulf of Mexico, Louisiana, coast and for Grays Harbor Entrance, Washington, USA. With the nonlinear wave-wave interaction, the calculated wave height along the Louisiana coast is approximately 10 percent

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higher than without the nonlinear energy transfer. For Grays Harbor, the nonlinear wave-wave interaction effect was investigated at four gauges, HMB1 to HMB4, of Half-Moon Bay in the lee of the south jetty where wave diffraction is strong (Figure 3). Figure 4 shows the comparison of calculated wave heights with and without the nonlinear energy transfer at HMB1 and HMB2. For these gauges, the calculated wave heights with the nonlinear energy transfer are typically 5 percent higher than without the nonlinear energy transfer. Additional comparisons of the calculated nonlinear wave energy transfer with data from other sites will be presented in the full paper.

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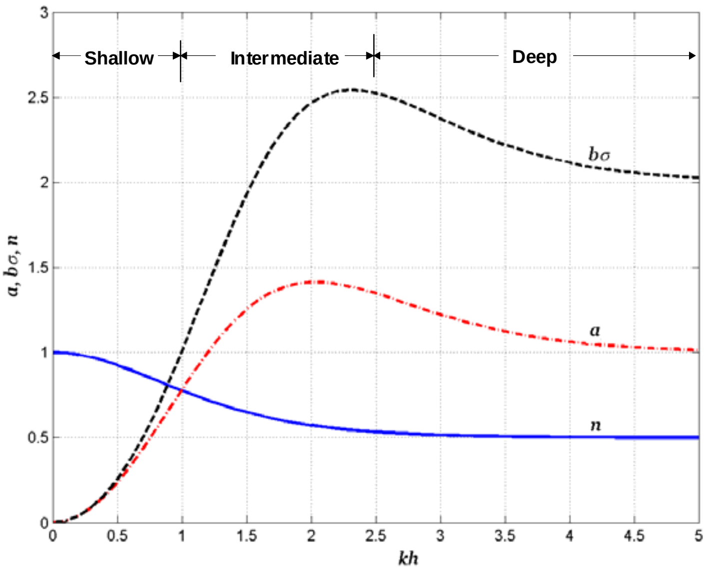

Figure 1. a, $b\sigma$, and n as functions of kh.

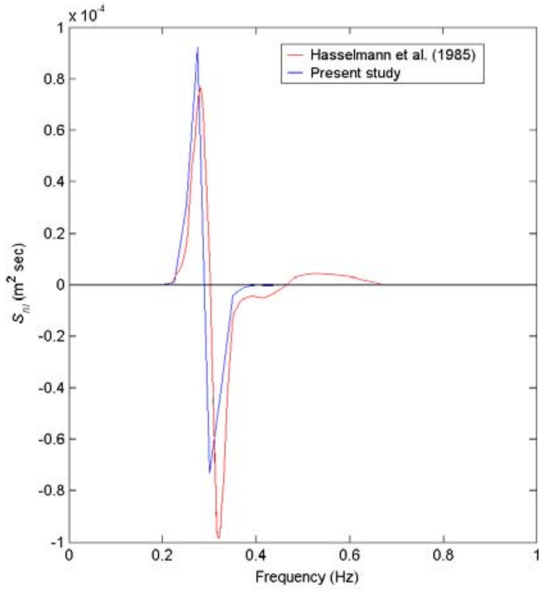

Figure 2. Calculated and exact S_{nl} for r = 5.

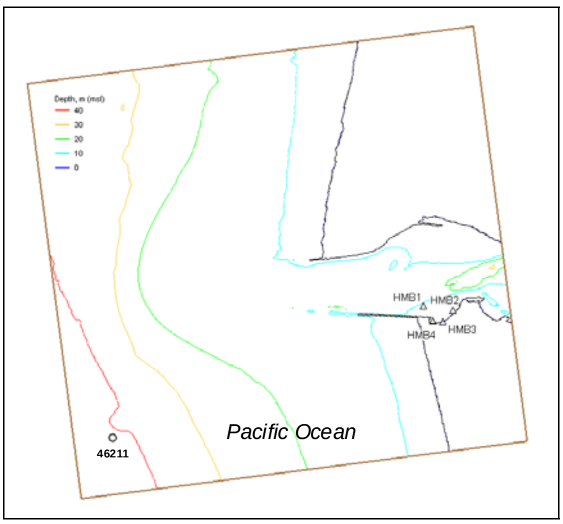

Figure 3. Grays Harbor gauge locations.

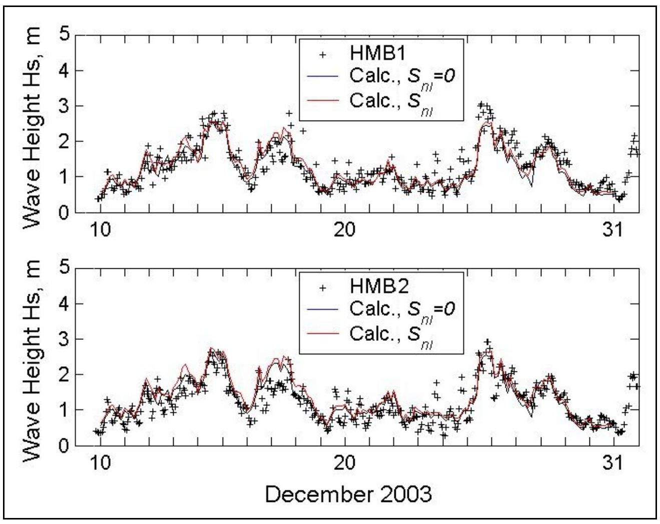

Figure 4. Measured and calculated waves, December 2003.